### NOTATION

I, current strength; P, pressure; G, gas flow rate; R and L, radius and length of the channel;  $V_z$  and  $V_r$ , axial and radial velocity components; E, electric field strength;  $T_e$  and  $T_h$ , temperatures of electrons and heavy particles;  $\Theta = T_e/T_h$ , degree of nonequilibrium;  $n_e$ ,  $n_i$ , and  $n_n$ , numbers of electrons, ions, and atoms per unit volume;  $J' = J - \Delta J$ , effective ionization potential;  $\Delta J$ , lowering of ionization potential in the plasma;  $\Gamma_i$ , change in the number of ions per unit volume;  $\rho$ , gas density;  $\sigma$ , coefficient of electrical conductivity;  $\lambda_e$  and  $\lambda_h$ , coefficients of thermal conductivity of electrons and heavy particles; k, Boltzmann number;  $\nu$ , collision frequency;  $m_e$  and  $m_n$ , masses of an electron and an atom; r and z, radial and axial coordinates;  $Z_i(T_e)$  and  $Z_n(T_e)$ , statistical sums over states of electronic excitation of ions and atoms.

#### LITERATURE CITED

- 1. V. N. Kolesnikov, Tr. Fiz. Inst., Akad. Nauk SSSR, Fiz. Opt., 30, 66-157 (1964).
- 2. A. I. Ivlyutin, in: Materials of the Seventh All-Union Conference on Low-Temperature Plasma Generators [in Russian], Part 2, Alma-Ata (1977), pp. 27-30.
- 3. V. M. Lelevkin and V. F. Semenov, in: Ninth All-Union Conference on Low-Temperature Plasma Generators [in Russian], Frunze (1983), pp. 24-25.
- 4. S. I. Braginskii, Vopr. Teor. Plazmy, No. 1, 183-272 (1963).
- 5. L. N. Panasenko and V. G. Sevast'yanenko, Inzh.-Fiz. Zh., <u>48</u>, No. 2, 279-284 (1985).
- 6. R. I. Soloukhin (ed.), Radiative Transfer in High-Temperature Gases (Handbook) [in Russian], Moscow (1984).
- 7. D. M. Chen, K. Hsu, and E. Pfender, Plasma Chem. Plasma Proc., 1, No. 3, 295-314 (1981).
- 8. L. N. Panasenko and V. G. Sevast'yanenko, Investigation of Processes in Plasma Heater
- Devices [in Russian], Minsk (1986), pp. 4-12.
- 9. A. V. Potapov, Teplofiz. Vys. Temp., 4, No. 1, 55-58 (1966).
- S. Veis, Czechoslovak Conference on Electronics and Vacuum Physics, Prague (1968), pp. 105-107.
- 11. D. Kannappan and T. K. Bose, Phys. Fluids, <u>20</u>, No. 10, 1668-1673 (1977).
- 12. D. M. Chen and E. Pfender, IEEE Trans. Plasma Sci., 9, No. 4, 265-274 (1981).

# STRUCTURE OF THE FIELD OF REFLECTED RADIATION FOR RADIATIVE

HEAT EXCHANGE IN SYSTEMS WITH SPECULAR SURFACES

A. V. Arendarchuk

UDC 621.365.24:536.2:535.312/313

Problems of the calculation of radiative heat exchange in systems with specular surfaces of arbitrary shape are considered.

In the design of modern electrothermal apparatus with specular surfaces (reflectors, mirrors) of complicated three-dimensional shapes one needs a highly informative method of calculating the distribution (field) of radiant flux in the radiation source-mirror-receiver system. Such a calculation allows one to estimate the effectiveness both of the chosen system as a whole and of its individual components [1].

Two main approaches to the solution of problems of the reflection of radiant flux in thermal apparatus are known. The first is to investigate the transfer of a portion (beam) of the self-emission of the source in the process of its successive reflections in the system; the second is to investigate the incident and effective fluxes of each surface of the system by setting up integral (or zonal) equations for them.

For systems with specular surfaces it is more appropriate to use the first approach in practical applications. This is explained by the correspondence of the calculation algorithm to the physical representation of the process of energy transfer in such systems:

All-Union Scientific-Research, Design-Construction, and Technological Institute of Electrothermal Equipment, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 4, pp. 617-621, April, 1987. Original article submitted March 4, 1986.



Fig. 1. Geometry of the "source-mirror-receiver" system (surfaces  $A_1$ ,  $A_2$ , and  $A_3$ , respectively) and elements of the structure of the calculated field (ray tubes) in this system.

Fig. 2. Caustic of a parabola (from [23]).

In specular reflection the radiation beam does not lose the properties of directivity, as happens in diffuse reflection, but propagates further along strictly determined trajectories (it is just this property that determines the functional purpose of reflectors: the formation of a radiant field of a given, often rather subtle and complicated structure).

The field structure in radiative heat exchange can be represented using various concepts employed in the analysis of radiant fluxes. Thus, in [2] the interpretation of radiant flux is connected with concepts of the radiation vector, indicating the direction of energy transfer, with its absolute magnitude determining the power of this transfer through a unit area perpendicular to the direction of the vector, and the line of radiant flux, at each point of which the radiation vector is directed along a tangent to it. In our problem, with a specular surface, it is more practical to use the concept of an elementary ray tube, formed by a beam of rays traveling close together, since in this case the solution has greater clarity and informativity. In particular, the shape of the caustic — one of the properties of a field of reflected radiation — is easily determined as the envelope of the reflected rays. Along with a beam of rays, we shall also consider their associated radiation wave front [3]. These two concepts are inseparable in the description of the field structure and energy transfer in the approximation of ray (geometrical) optics [4, 5]: The rays are defined as trajectories orthogonal to the wave fronts, and vice versa. The use of the wave front in the given problem allows us to generalize existing particular solutions.

Let us consider the distribution of reflected flux for the case when the space of radiation source-mirror-receiver system (in thermal apparatus this is the so-called "oven space") is filled with a nonabsorbing, nonscattering, and nonrefracting medium; then the rays filling the tube are straight. In Fig. 1 we show such a tube, starting at an element  $dA_1$  of the emitter (source) and ending at an element  $dA_2$  of the mirror (the direction of energy transfer is shown by an arrow). After reflection, the radiation beam will propagate within another tube, now starting at  $dA_2$  and ending at the receiver, "cutting out" an element  $dA_3$  on it.

As is well known [4], the relation

$$dE\left(\overline{r}_{dA_{i}}\right)dA_{i} = \text{const} \tag{1}$$

is valid for any cross section of a ray tube. From this we have

$$dE(\vec{r}_{dF_1}) dF_2 = \rho dE(\vec{r}_{dA_2}) dA_2 = \rho dE(\vec{r}_{dF_1}) dF_1, \qquad (2)$$

where  $dF_1$  and  $dF_2$  are elements, "cut out" by the tube, of the surfaces of wave fronts of the beam incident (initial) on the mirror and that reflected from it, respectively. If  $\ell$  is the distance between  $dA_2$  and  $dF_2$  along the tube, then

$$dE(\bar{r}_{dF_2}) = \rho dE(\bar{r}_{dA_2}) \frac{dA_2}{dF_2} = \rho dE(\bar{r}_{dA_2}) \frac{1}{|Kl^2 - 2Hl + 1|} ;$$
(3)

here the ratio of the surface elements (the divergence of the tube [6] or the exchange coefficient [3]) is revealed by methods of differential geometry. If  $dA_3$  is substituted in place of  $dF_2$ , i.e., the flux density on an element of the receiver is determined, then the cosine of the angle of incidence of the radiation on the receiver appears in Eq. (3) (see Eq. (5) in [3]).

The essence of revealing the ratio of elements in (3) consists in mapping an element of the mirror surface onto the surface of the receiver by the reflected wave front. In the physical interpretation this comes down to the change in the shape of the wave front, and hence in the cross section of the ray tube, in reflection. Of particular interest is the solution for an initial front of arbitrary shape, which makes it possible, by taking an element of the reflector as a secondary source, in particular, to extend the results obtained to the case of multiple reflections. The values of K and H, expressed through the curvature of the surfaces of arbitrary shape of the initial wave front and the mirror, are given in [7-9] and are used in [3] for problems of radiative heat exchange. The well-known relations for spherical [10-12] and plane [13] fronts can be obtained in the form of particular cases from (3) or from (5) in [3]. Equations for the multiple reflection of an initial spherical front are given in [14].

On the other hand, using the laws of ray optics, the field at the point  $r_{dF_2}$  can be expressed through the value of the field at the point  $r_{dA_2}$  of the mirror and the principal radii of curvature  $R_1$  and  $R_2$  of the reflected wave front [4, 15]. Under the conditions of our problem

$$dE(\bar{r}_{dR_2}) = \rho dE(\bar{r}_{dA_2}) \frac{R_1 R_2}{(R_1 - l)(R_2 - l)} = \rho dE(\bar{r}_{dA_2}) \left(\frac{l^2}{R_1 R_2} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)l + 1\right)^{-1}.$$
 (4)

Similar equations were given in [16, 17], for a plane front in the latter paper. The conversion from Eq. (4) to (3) is given by the relation [6, p. 116]

$$R_{1,2} = \frac{1}{K} (H \pm \sqrt{H^2 - K}).$$
 (5)

We note that from (3) or (4) for plane specular surfaces it is easy to obtain [18] the elementary resolvent angular coefficients, well known in the theory of radiative heat exchange, the apparatus of which has proved impossible to extend to curved surfaces of arbitrary shape (see the analysis of [19] in [10-12, 18]).

The shape and position in space of one of the distinctive features of the structure of the field of reflected radiation, the caustic [4, 5], are determined from (3) or (4) with  $\ell = R_1$  and  $\ell = R_2$ . In practical calculations it is of interest as the region where the density of radiant energy is extremely high. Thus, by a joint solution of the equations of the caustic and the receiver surface, it is easy to determine the location of points of increased flux density on the receiver. Knowledge of the properties of caustics acquires particular significance in the design of apparatus with several specularly reflecting surfaces, particularly with retroreflectors, where the caustics are a factor influencing the characteristics of the entire system.

One of the widely known examples is the caustic of a beam of parallel rays reflected from a spherical mirror (see, e.g., [6, 11, 12]). The shape of the caustic for such a popular type of mirror as parabolic mirrors is less well known. Thus, in [20, 21] the shape of this caustic was determined for oblique incidence of a beam of parallel rays on a paraboloid. This shape has long been known, however: See the references in [22], where it is also noted that "it has great similarity to the Cartesian surface" (Fig. 2).

The calculation method presented in the present article, at the basis of which lies the mapping of an element of the mirror onto the receiver by a reflected wave front, was realized in a three-dimensional mathematical model of an infrared exciter with a specular reflector [1].

In conclusion, we note that all the relations given above are also valid for tubes of finite cross section, but now with respect to the average values of the respective quantities. It is only necessary to allow for the fact that, in realizing the above method in mathematical models, the different approaches (tubes of elementary or finite cross sections) will correspond to similar algorithms, differing mainly in the means of calculating the flux at the receiver.

#### NOTATION

 $dA_i$ , an element of the i-th surface;  $dE(\bar{r}_{dA_i})$ , flux density in the vicinity of the point  $\bar{r} \in dA_i$ ;  $R_1$  and  $R_2$ , principal radii of curvature of the reflected front;  $K = 1/R_1R_2$ , total (Gaussian) curvature of the reflected front;  $H = 0.5((1/R_1) + (1/R_2))$ , average curvature of the reflected front;  $\rho$ , reflection coefficient of the specular surface.

#### LITERATURE CITED

- 1. A. V. Arendarchuk, Elektrotekh. Promst., Ser. Elektrotermiya, No. 9 (259), 2-3 (1984).
- Yu. A. Surinov, Heat Transfer and Thermal Modeling [in Russian], Moscow (1959), pp. 319-349.
- 3. A. V. Arendarchuk, Inzh.-Fiz. Zh., <u>50</u>, No. 3, 494 (1986).
- 4. M. Born and E. Wolf, Principles of Optics, 4th ed., Pergamon Press, Oxford-New York (1969).
- 5. O. N. Stavroudis, The Optics of Rays, Wavefronts and Caustics, London (1972).
- 6. Yu. A. Kravtsov and Yu. I. Orlov, Geometrical Optics of Nonuniform Media [in Russian], Moscow (1980).
- 7. V. A. Fok, Problems of Diffraction and Propagation of Electromagnetic Waves [in Russian], Sovetskoe Radio, Moscow (1970), pp. 155-178.
- 8. I. M. Fuks, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 8, No. 6, 1078-1088 (1965).
- 9. F. G. Bass and I. M. Fuks, Wave Scattering on a Statistically Uneven Surface [in Russian], Sec. 28 (1972), pp. 313-319.
- 10. D. G. Burkhard, D. L. Shealy, and R. U. Sexl, Int. J. Heat Mass Transfer, <u>16</u>, 271-280 (1973).
- 11. D. G. Burkhard and D. L. Shealy, J. Opt. Soc. Am., <u>63</u>, No. 3, 299-304 (1973).
- 12. D. L. Shealy and D. G. Burkhard, Opt. Acta, <u>20</u>, No. 4, 287-301 (1973).
- 13. A. T. Vlasov, I. A. Vatutin, and I. V. Skutova, Processes of Heat and Mass Exchange in Elements of Thermooptical Devices [in Russian], Minsk (1979), pp. 46-49.
- 14. D. L. Shealy and D. G. Burkhard, Opt. Acta, 22, No. 6, 485-501 (1975).
- 15. B. Ya. Gel'chinskii, Dokl. Akad. Nauk SSSR, 118, No. 3, 458-460 (1958).
- 16. G. L. Voronkov, Svetotekhnika, No. 1, 19-20 (1973).
- 17. N. N. Ermolinskii and V. D. Komissarov, Byull. Vses. Elektrotekh. Inst., No. 2, 21-26 (1934).
- 18. D. G. Burkhard and D. L. Shealy, Int. J. Heat Mass Transfer, <u>16</u>, 1492-1496 (1973).

19. J. A. Plamondon and T. E. Horton, Int. J. Heat Mass Transfer, 10, 665-679 (1967).

- 20. J. B. Scarborough, Appl. Opt., <u>3</u>, No. 12, 1445-1446 (1964).
- 21. H. J. Stalzer, Appl. Opt., 4, No. 9, 1205-1206 (1965).
- 22. A. Sonnefeld, Concave Mirrors [Russian translation], V. Fabrikant (ed.), Moscow-Leningrad (1935).

## NONLINEAR EFFECTS DURING FILTRATION IN BEDS WITH

#### LARGE-SCALE STRUCTURAL INHOMOGENEITY

Yu. A. Buevich and V. A. Ustinov

UDC 532.546.7:622.276.3

A model of the nonlinear hydraulic relation between the porous volumes of a significantly inhomogeneous bed is proposed, and its influence on the draining of unit volume of borehole, as well as on the curves of bed-pressure recovery and on the indicator diagrams of the borehole, is investigated.

Recently, the industrial petroleum content of distinctive clay bituminous rocks with unusual (in comparison with well-known collectors) and in many respects unique properties has been established; the collector of the so-called Bazhenov formation of western Siberia

A. M. Gor'kii Ural State University, Sverdlovsk. West-Siberian Scientific-Research Geological and Prospecting Petroleum Institute, Tyumen'. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 4, pp. 621-633, April, 1987. Original article submitted February 27, 1986.